Reductions for q-Hypergeometric Terms

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The q-analogue of hypergeometric terms, called q-hypergeometric terms, is a class of ubiquitous special functions in the study of q-combinatorial identities, integer partitions, modular forms and quantum groups. A basic problem concerning q-hypergeometric terms is the q-summability problem, that is, to decide whether or not a given q-hypergeometric term has a q-hypergeometric indefinite sum (also called antidifference). The q-analogue of Gosper's algorithm solves this problem completely, in the sense that it either finds a q-hypergeometric indefinite sum of the given q-hypergeometric term (in this case we call the given term q-summable), or denies the existence of such indefinite sums. In order to get more information about the additive structure of q-hypergeometric terms, we translate the idea of the modified Abramov-Petkovšek reduction for hypergeometric terms into the q-hypergeometric setting, and then develop an analogous reduction algorithm. This q-reduction algorithm decomposes a q-hypergeometric term into the sum of two terms, in which the first summand is q-summable, and the second one is not unless it is zero. In particular, it solves the q-summability problem as well. Moreover, this q-reduction algorithm can be further used to compute the telescopers for bivariate q-hypergeometric terms.